Technical Comments

Comment on "Prediction of Subsonic Aerodynamic Characteristics: A Case for Low-Order Panel Methods"

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MASKEW has recently presented the attractive features of two low-order panel methods for complex three-dimensional configurations.[1] He has demonstrated that these low-order methods are inherently more efficient than the higher-order formulations.

Both methods appear to be robust with respect to body shape, but not with respect to panel distribution. Indeed, Maskew observed that, in the case of airfoils, the "simpler" method is not sufficiently forgiving when faced with bad panel distributions. More precisely, the pressure distribution near the trailing edge of a NACA 0012 section at $\alpha = 10$ deg was found to be significantly inaccurate in the case of asymmetrical panelling (i.e., more panels on the upper than on the lower side of the airfoil). Since the other formulation had no significant problems, it has been accepted as the superior method.

The above experiment has motivated the following analysis, which restores the superiority of the "simpler" method by a suitable modification of the integral equation.

It is convenient to write the equations involved in the analysis for steady potential flow in the region R_+ around a rigid, impermeable, nonlifting body R_- bounded by the surface S. If ϕ_∞ denotes the potential of the freestream velocity $V_\infty = \nabla\phi_\infty$, then the disturbance potential ϕ is the solution of the exterior Neumann problem:

$$\Delta\phi(P) = 0 \qquad (P \text{ in } R_+)$$

$$\frac{\partial\phi_+(p)}{\partial n_p} = -\frac{\partial\phi_{\infty+}(p)}{\partial n_p} \qquad (p \text{ on } S)$$

$$\lim\phi(P) = 0 \qquad (|P| \to \infty)$$

In the method[1,2] finally accepted by Maskew, ϕ is represented by Green's exterior formula as a sum of two potentials generated by a distribution of sources and doublets on S, respectively:

$$\phi(P) = \frac{1}{4\pi}\int_S [n_q \cdot V_{\infty+}(q)]\left(\frac{1}{|P-q|}\right)\mathrm{d}S_q + \frac{1}{4\pi}\int_S \phi_+(q)\frac{\partial}{\partial n_q}\left(\frac{1}{|P-q|}\right)\mathrm{d}S_q \tag{1a}$$

The density ϕ_+ of the doublet distribution is the value of ϕ on S and is the unique solution of the integral equation:

$$\phi_+(p) - \frac{1}{2\pi}\int_S \phi_+(q)\frac{\partial}{\partial n_q}\left(\frac{1}{|p-q|}\right)\mathrm{d}S_q = \frac{1}{2\pi}\int_S [n_q \cdot V_{\infty+}(q)]\left(\frac{1}{|p-q|}\right)\mathrm{d}S_q \tag{1b}$$

In the "simpler" method[1,3] considered, ϕ is represented by the potential of a doublet distribution:

$$\phi(P) = \frac{1}{4\pi}\int_S \Phi_+(q)K(q,P)\,\mathrm{d}S_q \tag{2a}$$

(This representation is possible because ϕ_∞ is a harmonic function in the region $R_- + S$ occupied by the body.) $K(q,P)$ is the factor of $\phi_+(q)$ in the second integral in Eq. (1a). Now the density Φ_+ of the distribution is the value of the total potential $\Phi = \phi_\infty + \phi$ on S and is the unique solution of the integral equation:

$$\Phi_+(p) - \frac{1}{2\pi}\int_S \Phi_+(q)K(q,p)\,\mathrm{d}S_q = 2\phi_{\infty-}(p) \tag{2b}$$

The attractively simple right-hand side of this equation will be recognized as a Trojan horse—a dangerous gift. The subscript (-) may be dropped since $\phi_{\infty-}(p) = \phi_{\infty+}(p)$.

Let us now analyze heuristically these two methods in the light of Maskew's experiment.

We observe that the left-hand sides of the integral equations (1b) and (2b) contain the same integral operator. This implies that the linear algebraic systems, resulting from the discretization of S by a finite number of panels, have the same coefficient matrix (influence matrix). Only the right-hand sides of Eqs. (1b) and (2b) are different, leading to different solution vectors. If, in Maskew's experiment, the solution based on Eq. (2b) appears to be inaccurate while the solution based on Eq. (1b) is acceptable, then the inaccuracy can be caused only by the term $2\phi_\infty$. The corresponding vector may point in a less favorable direction with respect to the eigenvectors of the influence matrix than the inhomogeneity vector of Eq. (1b). We have an example of the fact that the condition of a linear system depends on both the matrix and the inhomogeneity vector. However, it is sufficient to know that we have to turn the "vector" $2\phi_\infty$ in a more favorable direction. This is rather simple to realize. We write Eq. (2b) in the *mathematically equivalent* form:

$$\phi_+(p) - \frac{1}{2\pi}\int_S \phi_+(q)K(q,p)\,\mathrm{d}S_q = \phi_\infty(p) + \frac{1}{2\pi}\int_S \phi_\infty(q)K(q,p)\,\mathrm{d}S_q \tag{2c}$$

This equation is *numerically superior* to Eq. (2b) and even Eq. (1b). Indeed, the left-hand sides of Eqs. (lb) and (2c) are identical and the right-hand sides are equal by virtue of Green's formula for ϕ_∞. Since Eq. (1b) is well-behaved in the experiment, Eq. (2c) appears to be a good guess. The integrals

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at the right-hand side of Eq. (2c) have the same form as those on the left-hand side and can therefore be evaluated as safely and accurately as in Eq. (1b). Incidentally, these integrals (doublet potentials) lead to simpler expressions than those on the left-hand side of Eq. (1b) (See the explicit formulas in Ref. 2.) Moreover, in any low-order implementation, the integrals on the right-hand side of Eq. (2c) have already been computed to yield the influence matrix and can simply be called from computer memory. We conclude that the modified integral equation (2c) is preferable to Eq. (1b). This is also true in the case of lifting bodies, since the additional integral over the wake is the same in both Eqs. (1b) and (2c). [See Ref. 1 for the details. There is a additive term $4\pi\phi_{\infty j}$ missing on the right-hand side of Eq. (5).]

Finally, we note that the doublet-only formulation of Eq. (2a)—regardless of the particular integral equation chosen—is less general than the formulation of Eq. (1a) based on the full Green's formula. The former approach is possible only if ϕ_∞ exists and is given at least on S. For the latter approach, only V_∞ on S is required. This is convenient in applications in which ϕ_∞ does not exist or is difficult to obtain explicitly.[4] Of course, it is not hard to incorporate both Eqs. (1b) and (2c) in the same code and to use Eq. (2c) whenever ϕ_∞ is available.

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References

[1]Maskew, B., "Prediction of Subsonic Aerodynamic Characteristics: A Case for Low-Order Panel Methods," *Journal of Aircraft*, Vol. 19, Feb. 1982, pp. 157-163.

[2]Morino, L., Chen, L. T., and Suciu, E. O., "Steady and Oscillatory Subsonic and Supersonic Aerodynamics around Complex Configurations," *AIAA Journal*, Vol. 13, March 1975, pp. 368-374.

[3]Groh, G., "Eine Integralgleichungsmethode für die räumliche Potentialströmung um beliebige Körper," *Journal of Applied Mathematics and Physics (ZAMP)*, Vol. 32, Jan. 1981, pp. 107-109.

[4]Chorin, A. J., "Numerical Study of Slightly Viscous Flow," *Journal of Fluid Mechanics,* Vol. 57, No. 4, 1973, pp. 785-796.

Reply by Author to G. Gy. Groh

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GROH'S comments on the low-order panel method formulation are very useful from a mathematical viewpoint, but it should be emphasized that the methods discussed in the paper are just two of a large number of possible internal potential flows. For example, in the earlier work an internal flow parallel to the wing chord plane, i.e., $\phi_i = -x$ was also considered and showed a slight improvement over method 2 in the trailing-edge region. The main function of a "good" internal flow would appear to be to minimize the perturbation required of the doublet solution to satisfy the boundary condition. However, the formulation of such an internal flow may not always be convenient. As Groh points out, method 1 described in the paper has a simpler formulation than method 2; in fact, it probably has *the* simplest formulation of any method providing the lifting solution. To emphasize this point, method 1 uses just one term from the three-component velocity influence coefficient in the original *nonlifting* Douglas-Neumann code. However, it has three drawbacks:

1) The doublet value being the external *total* potential can lead to increased numerical error in the solution.

2) Obtaining velocities by numerical differentiation of the *total* potential is prone to error (in method 2 only the gradient of the *perturbation* potential is obtained numerically).

3) Nonzero normal velocities cannot be treated on the closed boundaries—source singularities are required to cancel the jump in normal velocity across the boundary.

As Groh points out, drawbacks 1 and 2 can be removed. A general way of achieving this is to separate the doublet distribution into two (or more) parts: an applied part, for which the velocity is known, and a small unknown part, which is to be solved. Again, there are a number of possible combinations; the obvious choice is to use ϕ_∞ as the applied distribution and to solve for the perturbation potential. This leads to Groh's Eq. (2c). The solved part of the doublet distribution (i.e., the perturbation potential) is then numerically the same as for method 2. The gradient of the *perturbation* potential is then evaluated numerically and added to the known local tangential component of V_∞.

Drawback 3 is the main reason for preferring the method 2 formulation for the general case. The power of the panel method lies in representing complete aircraft configurations, including modeling of the inlet flow, jet efflux, boundary-layer displacement effect, unsteady motions, and perturbation solutions. These all lead to the need to include the source term for the general case. As Groh observed, the two forms can be mixed in a given problem; in fact, in some complex cases, the ideal setup would be to have a number of internal flows for application in different parts of the problem to minimize the magnitude of the local doublet solution.

Finally, Groh has pointed out the missing $4\pi\phi_{\infty J}$ in Eq. (5) of the paper; we should also note that the last term in Eq. (3) should be $4\pi\phi_{\infty P}$ rather than $\Phi_{\infty P}$.

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Comment on "Effects of Atmospheric Turbulence on a Quadrotor Heavy-Lift Airship"

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THE subject paper[1] reports a study of the response of a particular LTA to atmospheric turbulence, utilizing a model and method of analysis developed in Refs. and 2 and 3. Unfortunately, there is a theoretical error in the formulation of forces caused by fluid acceleration that leads to an overestimation of the response (loads and motions). There is another flaw in the analysis—a bad assumption—that works in the opposite direction. These two points are elaborated below.

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